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Constructing Coincident and Leading Indices of Economic Activity for the Brazilian Economy

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Abstract

This paper has three original contributions. The first is the reconstruction effort of the series of employment and income to allow the creation of a new coincident index for the Brazilian economic activity. The second is the construction of a coincident index of the economic activity for Brazil, and from it, (re) establish a chronology of recessions in the recent past of the Brazilian economy. The coincident index follows the methodology proposed by TCB and it covers the period 1980:1 to 2007:11. The third is the construction and evaluation of many leading indicators of economic activity for Brazil which fills an important gap in the Brazilian Business Cycles literature.

Keywords: Coincident and Leading Indicators, Business Cycles, Common Features, Latent Factor Analysis

J.E.L. Codes: C32, E32.

1 Introduction

An important concern of any modern society in what is the current "state" of economy and what should be the state of the economy in the near future. Entrepreneurs and individuals are interested in the question because their profits and welfare are, respectively, a function of it. Governments also have an interest in the subject for budgetary and welfare issues. Unfortunately, no one possesses a series that represents the "state of the economy" because it is a latent variable, i.e., it is non-observable.

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Stock and Watson (1999) argue that, if we were to choose one variable to best represent the state of the economy, this variable would be the Gross Domestic Product (GDP). They claim that "[...] fluctuations in aggregate output are at the core of the business cycle so the cyclical component of real GDP is a useful proxy for the overall business cycle [...]". However, GDP is not readily available without measurement error, making it of little use for decision making in this context. The idea of bringing together information on GDP to construct coincident and leading indices for the U.S. is also present in Mariano and Murosawa (2003).

Including alternative information to estimate the state of the economy is also present in the recent effort of Issler and Vahid (2006). They argue that current U.S. research misses a vital piece of information on the state of the economy – the NBER dating committee decisions. They claim that, if "we are asked to construct an index of the health status of a patient, [and] we know that the best indicator of the health of the patient is the results of a blood test, [but] blood samples cannot be taken too frequently, and test results are only available with a lag, sometimes too long to be useful, [making our index] a function of variables such as blood pressure, pulse rate and body temperature that are readily available at regular frequencies. In order to estimate the best way to combine these variables into an index, would we (i) use the historical data on these variables only, or, (ii) use the historical blood test results as well? The answer is, obviously, the latter." Here, blood-test results play a similar role to the NBER dating committee decisions.

The lack of a direct measure of the state of the economy has led to the construction of proxies that can be used in real time. These are the so-called coincident indices of economic activity. From them we can also construct leading indices of observables that help predicting the current state of the economy – the so-called leading indices of economic activity.

With the exception to the work of Contador (1977) and Contador and Ferraz (1999), research on coincident and leading indices of economic activity in Brazil is fairly young and most of the literature dates from the 2000's. Chauvet (2001) and Picchetti and Toledo (2002) and use common-factor models to generate a monthly coincident indicator of economic activity. Chauvet (2002) uses a two-state Markov Chain characterizing a recession or an expansion to propose a chronology for Brazilian business cycles. On a broader study, Duarte, Issler and Spacov (2004) evaluated three candidates for composite coincident indices: The Conference Board's (TCB's) index; Spacov's (2000) index, and Issler and Vahid's (2006) index. Using quadratic loss, the dating of these three indices was compared with that of a monthly proxy of Brazilian GDP, suggesting that the Brazilian coincident index should use the methodology put forth by TCB.

Unfortunately, part of this recent research effort in Brazil came to a halt because of the recent redesign of the official employment survey conducted by IBGE – Monthly Employment Survey (*Pesquisa Mensal do Emprego*) – which provides monthly Brazilian data on employment and labor income. Indeed, the change in the survey design in 2002 is so drastic that it eliminates long-span time-series on employment and income, which are crucial series for business-cycle research using TCB- and NBER-oriented methods.

The first goal of this paper is to resume business-cycle research in Brazil using these methods, which proved to be valuable after the empirical results in Duarte, Issler and Spacov. Indeed, one of the main challenges of Brazilian business-cycle research is to back-cast currently available income and employment series to be able to form a long enough coincident index with the usual series used in TCB's method – industrial production, sales, income and employment. Here, we devote and a great deal of effort in reconstructing employment and income using a novel State-Space representation. It is based on the interpolation method proposed by Mönch and Uhlig (2005): a very flexible setup that allows the estimation of a wide range of models. As usual, estimation of the unobserved components in these models is performed employing the kalman filter.

Once we obtain a long enough span of the usual series used in TCB's method, we compute a new composite coincident index of Brazilian economic activity. Its dating of recessions is compared with those in Duarte, Issler and Spacov and with those implied by the monthly GDP estimate computed by Issler and Notini (2008).

Our last contribution is regarding the construction of leading indices of economic activity to track the composite coincident index proposed here. Although coincident indices have been relatively well studied in Brazil, leading indices have not. In constructing leading indices we take into account three interesting and novel features in Brazilian business-cycle research: (i) we consider using Granger (1969) causality tests, as well as novel alternative criteria in choosing candidate series to be included in leading composite indices; (ii) we investigate the ability of survey-based time series to lead our composite index; and, (iii) we compare the survey-based composite leading indices with standard leading indices.

Although comparisons are based on a variety of features of the dating properties of these different indices, our decision to validate the current composite index is mostly based on a variant of the QPS quadratic-loss statistic proposed by Diebold and Rudebusch (2001).

Empirical results obtained here are compared with the previous literature on Brazil. In evaluating different results and techniques used in constructing coincident and leading indices, we borrow from the almost century-long debate on this issue that has been present in the U.S. economy, and a similar half-century or older debate in Europe.

This article is organized as follows. Section 2 contains a brief review of the international and the Brazilian literature. Section 3 presents the Kalman filter model. Section 4 presents the data and the main results. Section 5 concludes.

2 Literature Review

2.1 The International Experience

There has been a fair amount of research on cyclical indicators since the pioneering work of Arthur F. Burns and Wesley Mitchell, which lead to their classic book on business cycles – Burns and Mitchell (1946). Their work has led to the construction of composite indices of leading, coincident, and lagging indicators of economic activity. While their research on the subject was focused on the U.S. economy, it soon become apparent that these methods had the potential to be applied on what we now label a "global scale." Indeed, European research based on their methods gained momentum after WW-II, while the same happened in Latin America after inflation stabilized in the region by the second half of the 1990's.

The National Bureau of Economic Research (NBER) was founded in 1920 and started the work of dating the U.S. business cycles very early in the 20th Century. They are responsible for the development of methods detection the turning points in the level of an economic series (or in its logs) – classical business-cycle analysis – and for the detection of turning point on an isolated cyclical component (a detrended series) – growth-cycle analysis.

The NBER Business-Cycle Dating Committee is responsible for the U.S. business cycles dating since 1978. The most educated estimate of U.S. turning points is embodied in the binary variable announced by the NBER Business Cycle Dating Committee. The NBER Dating Committee summarizes its deliberations as:

> "The NBER does not define a recession in terms of two consecutive quarters of decline in real GNP. Rather, a recession is a recurring period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy."
> (Quoted from http://www.nber.org/cycles.html)

The problem with the NBER committee deliberations is its lag – usually six months to one year after a turning point has occurred. This makes it of little practical use for instant or direct decision-making purposes. The final decision is a consensus between different visions of the experts present in the Dating Committee meeting (a total of 7 experts on business-cycle dating). These deliberations can be viewed as a result of a survey involving a group of very educated business-cycle researchers. It is exactly this character that makes it an interesting variable for the purposes of CIRET.

The first constructed coincident index of U.S. economic activity was implemented by the Census Bureau, a task that was later transferred to The Conference Board (TCB) – a non-profit private entity whose main purpose is to do research on this field. Since 1995, by order of the Department of Commerce of the U.S., TCB established a series of leading, coincident, and lagging indicators of economic activity. The coincident indicator is an average of the four coincident series – production, income, sales and employment. TCB uses a simple average of the standardized differenced (logged) series. which is a way of treating equally the fluctuations of all four series in computing the index. TCB approach is somewhat heuristic, since it requires no estimation of a formal econometric model. Despite that, it works surprisingly well in practice; see the comparison in Issler and Vahid (2006) using the TCB index and alternative econometric-based indices in trying to replicate the NBER dating decisions.

As an alternative to heuristic methods such as TCB's, several authors have proposed methods of building indices supported by sophisticated econometric and statistic techniques. Stock and Watson (1998a, 1998b, 1998c, 1989, 1993a) were the first to apply the tools of modern time-series econometrics to build an approach able to construct leading and coincident indices; to detect turning points of economic activity; and to predict the probability of a recession. Their models formalize the idea that the reference cycle is best measured by looking at co-movements across several aggregate time series, making their experimental index an estimate of the value of a single unobserved variable – "the state of the economy". The observable variables used in estimating the state of the economy are the usual coincident series: industrial production, income, sales and employment, which are forecast employing additional leading series.

An important empirical drawback in Stock and Watson's approach was its failure to detect the U.S. recession in 1990-1991. Many papers tried to improve on Stock and Watson's method, while keeping the formal building block of a structural econometric model. We review here just a few. Forni et al. (2000) proposed an alternative approach to Stock and Watson's which is very close to the latter in spirit. In its more recent versions these authors build a dynamic common-factor model instead of a static one, i.e., based on current and lagged coincident series, not just current coincident series.

Chauvet (1998) improved on Stock and Watson's model with the inclusion of regime switching as proposed by Hamilton (1989). The idea is to capture asymmetries between expansions and contractions of the economic activity. It relies in the fact that contractions are more abrupt and shorter than expansions. Mariano and Murasawa (2003) extended Stock and Watson model in order to allow the use of mixed-frequency series, where GDP (quarterly measured) plays a central role. The coincident index is now the common factor of all four coincident series and also to interpolated monthly GDP, a sub-product of the analysis.

Finally, Issler and Vahid (2006) have a structural model for the NBER decisions, where the unobserved "state of the economy" is a function only of the cyclical behavior of the coincident series. They used canonical correlations analysis to filter out the noisy information contained in the usual four coincident series, building a composite coincident index that is matched to fit the information of the NBER decisions. Weights are estimated via an instrumental-variable Probit regression, which is then used to construct optimal coincident and leading indices (optimal 1-step ahead forecasts).

2.2 The Methodology of TCB

The ideas behind TCB's method are twofold: simplicity and robustness. Simplicity is used because they weight information in coincident and leading indices with equal weights, once one controls for the fact that different signals carry different information depending on their variance. One simple way to treat every series equally in this context is to standardize them, treating equally the standardized series. Robustness comes into play here, since standardizing is a way of robustly treating different realizations of the same random variable.

The coincident series is an equally-weighted linear combination of four coincident series (income (I_t), output (Y_t), employment (N_t), and sales (S_t)) once we control for the fact that the growth rate of these series have different variances. Hence, the coincident indicator uses weights constructed as:

$$\Delta \ln (CI_t) = \frac{1}{4}\left[\frac{\Delta \ln (I_t)}{\sigma_{\Delta \ln (I)}} + \frac{\Delta \ln (Y_t)}{\sigma_{\Delta \ln (Y)}} + \frac{\Delta \ln (N_t)}{\sigma_{\Delta \ln (N)}} + \frac{\Delta \ln (S_t)}{\sigma_{\Delta \ln (S)}}\right], \tag{1}$$

where $\sigma_{\Delta \ln (I)}$, $\sigma_{\Delta \ln (Y)}$, $\sigma_{\Delta \ln (N)}$, and $\sigma_{\Delta \ln (S)}$ are respectively the standard deviations of income, output, employment, and sales growth. It is straightforward to construct the level series $\ln (CI_t)$ or CI_t once we posses $\Delta \ln (CI_t)$.

The leading series are usually chosen because they have *turning points* that happen *before* those of the level series $\ln(CI_t)$ or CI_t. To determine that, we first need a definition of "turning points" and of "before." In this literature, turning points are usually determined using an accepted algorithm for turning points or local minima and maxima of a time series – the Bry-Boschan algorithm, Bry and Boschan (1971). With turning points of the target variable and of the potential leading series in hand, all we have to determine is whether those of the potential leading series precede those of the target series, something a simple average of peaks and throughs precedence can determine. Leading series are those that downturn or upturn prior to the target series, on average. Once we determine the candidates of leading series, all we have to do is to combine them. Again, the TCB's methodology uses simplicity and robustness: all leading series are combined using a procedure similar to (1).

2.3 The Brazilian Experience

Contador (1977) was the first author to develop Brazilian coincident and leading indices of economic activity. He employs a myriad of methods, although has an intensive use of principal-component analysis. Alternatively, Spacov (2000) and Issler and Spacov (2000) use canonical correlation analysis to the same end, where the latter method solves the usual problem of "scale indeterminacy" found in principal-component analysis.

Chauvet (2001) uses principal-component analysis. To generate a monthly coincident indicator and an estimate of the probability of a recession in Brazil. Chauvet (2002) models the innovation in trend of Brazilian GDP as a two-state Markov Chain characterizing a recession or an expansion. In these two papers, she offers a chronology of Brazilian recessions. Picchetti and Toledo (2002) only take industrial production into account to propose a common-factor model for Brazilian (industrial) production. The unobserved component is estimated using the kalman filter, along the lines of Stock and Watson and Forni et al. (2000).

More recently, Duarte, Issler and Spacov (2004) evaluated three alternative coincident-index methods of economic activity for Brazil: TCB's index, whose instantaneous growth rate is a equally weighted combination of the standardized growth rate of the four coincident series (output, income, employment, and sales); Spacov's (2000) index, whose instantaneous growth rate is a weighted combination of the growth rate of the four coincident series, where canonical correlations are used to form weights; Issler and Vahid's (2006) index, whose instantaneous growth rate is a weighted combination of the growth rate of the four coincident series, where IV-Probit-regression coefficients are used to compute coincident-series weights. Using quadratic loss, the dating of these three indices was compared with that of a monthly proxy of Brazilian GDP. The results suggest that the Brazilian coincident index should follow the methodology put forth by the TCB. Finally, based on this result, these authors propose a chronology of recessions for the Brazilian economy in the recent past.

A common problem in Brazilian statistical data is the constant revisions they are subjected to. In most instances, these revisions did not prevent the construction of a chained series. However, in 2002, the new redesign of *Pesquisa Mensal do Emprego (Monthly Employment Survey)* lead to a virtual discontinuity in the employment and income (labor income) series. Since these were two completely different survey designs, chaining the previous series with the new ones was not an option. This implied a halt in business-cycle research in 2002, unless we could back-cast the current series yielding a long enough time-series span for the study of business cycles. Indeed, this is exactly what we discuss next. In some sense, the current paper is an attempt to restart Brazilian business-cycle research post 2002, where newly reconstructed series are used to re-evaluate previous findings.

3 Back-Casting Using the Kalman Filter

In this section, we give a brief review of the Kalman filter model applied to back-cast two of our coincident series – employment and income. A detailed description of this technique can be found in Harvey (1989) or in Hamilton (1994).

Consider a vector of $n \times 1$ observables in period t – y_t, a $r \times 1$ vector of latent variables (non-observables) in period t – ξ_t, and a $k \times 1$ vector of predetermined variables in period t – x_t. A state-space representation is a way of summarizing the relationships between these 3 sets of variables, where the dynamic nature of the system is taken into account. In most applications, the state-space representation is linear, which leads naturally to the conditional log-likelihood of the system under Gaussian innovations and into a way of estimating the latent variables in the system. The latter is usually the ultimate goal of constructing such models.

The state-space representation considered here has a *state equation* and a *measurement equation*, respectively as follows:

$$\xi_{t+1} = \mathbf{F}\xi_t + v_{t+1} \quad (2)$$

$$y_t = \mathbf{A}'x_t + \mathbf{H}'\xi_t + w_t, \quad (3)$$

where $\mathbf{F}$, $\mathbf{A}'$, and $\mathbf{H}'$ are fixed coefficient matrices in this simplified setup, but could be time-varying in more elaborate applications. Indeed, we will make $\mathbf{H}'$ a time-varying matrix in back-casting employment and income for Brazil.

The state equation (2) describes the dynamics of the *state vector* (ξ_t) containing the latent variables we want to estimate. The observation equation (3) links the vector containing the observables y_t to the vector containing the pre-determined variables and the latent variables in the system.

The disturbances v_t and w_t are assumed to be orthogonal at all leads. Moreover, these error terms have a multivariate Normal distribution as follows:

$$\begin{pmatrix} v_t \\ w_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{pmatrix}\right), \tag{4}$$

which makes (2) and (3) to be a Gaussian conditional (linear) system in which estimation and forecasting can be based upon. The statement that x_t is predetermined (or "exogenous") means that x_t provides no information on v_{t+s} and w_{t+s}, $s \geq 0$, beyond that contained in $y_{t-1}, y_{t-2}, \cdots, y_1$. The coefficients matrices $\mathbf{F}$, $\mathbf{A}'$, and $\mathbf{H}'$, and the two variance-covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ can be estimated by maximizing the conditional log-likelihood function of the system, given initial conditions on $\xi_{1|0}$ and on its variance-covariance matrix, labelled $\mathbf{P}_{1|0}$.

We are interested in the values of the unobserved state variable – ξ_t. We can forecast them based on the full set of data, which is called the smoothed estimate of ξ_t, or, we can forecast ξ_t using only data up to period $t-1$, which is called the filtered estimate. Both are presented, respectively, below:

$$\xi_{t|T} = \mathbb{E}\left(\xi_t \,| y_1, x_1, \cdots, y_T, x_T\right), \tag{5}$$

$$\xi_{t|t-1} = \mathbb{E}\left(\xi_t \,| y_1, x_1, \cdots, y_{t-1}, x_{t-1}\right). \tag{6}$$

Our starting point in using the kalman filter to back-cast the employment and income is the paper by Möch and Uhlig (2005), where they used the filter to interpolate GDP from quarterly to monthly frequency. They assume that unobserved monthly GDP (labelled as y_t^+ here) follows an $AR(p)$ process explained by the exogenous regressors x_t and an $AR(1)$ error term:

$$\begin{aligned} \left(1 - \phi_1 L - \cdots - \phi_p L^p\right) y_t^+ &= x_t \beta + u_t \\ u_t &= \rho u_{t-1} + \varepsilon_t. \end{aligned}$$

They set the observed quarterly GDP (labelled as y_t here), simply as:

$$y_t = \sum_{i=0}^{2} y_{t-i}^+, \qquad t = 3, 6, 9, 12, \ldots \tag{7}$$

$$y_t = 0, \qquad \text{otherwise.} \tag{8}$$

Hence, quarterly GDP, which we can only observe on months $t = 3, 6, 9, 12$, is the sum of the corresponding monthly GDPs in that quarter. Otherwise, it is just zero. Notice that setting $y_t = 0$ for the months we do not observe GDP is a clever way of making quarterly GDP observable at the monthly frequency. The aggregation of monthly GDP can also be made averaging the y_t^+'s, i.e., as $y_t = \frac{1}{3}\sum_{i=0}^{2} y_{t-i}^+$.

If we assume that the polynomial $\left(1 - \phi_1 L - \cdots - \phi_p L^p\right)$ is of order one, i.e., $p = 1$, with coefficient ϕ, the state-space form of Möch and Uhlig's problem is the following:

$$\xi_t = \begin{pmatrix} y_t^+ \\ y_{t-1}^+ \\ y_{t-2}^+ \\ u_t \end{pmatrix} = \begin{pmatrix} \phi & 0 & 0 & \rho \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} y_{t-1}^+ \\ y_{t-2}^+ \\ y_{t-3}^+ \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} x_t\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ 0 \\ \varepsilon_t \end{pmatrix} \tag{9}$$

$$y_t = \mathbf{H}_t' \xi_t, \tag{10}$$

where (9) and (10) are respectively the state and the observation equations and the matrix $\mathbf{H}_t'$ is time-varying, with the following format:

$$\mathbf{H}_t' = \begin{cases} \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}, & t = 3, 6, 9, 12, \ldots \\ \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, & \text{otherwise.} \end{cases} \tag{11}$$

One interesting feature of the approach in Mönch and Uhlig is that it encompasses several data interpolation models that are state-space based, summarized in Table 1 below:

Table 1 – Resulting Model as a Function of ϕ and ρ in (9)

Model	ϕ	ρ
Static model in levels with IID residuals	0	0
Static model in levels with AR(1) residuals (Chow and Lin, 1971)	0	free
Static model in 1st differences with IID residuals (Fernandez, 1971)	0	1
Dynamic model in levels with IID residuals (Mitchell et al., 2005)	free	0
Dynamic model in 1st differences with IID residuals	free	1
Dynamic model in levels with AR(1) residuals	free	free

To assess the quality of interpolation, Mönch and Uhlig follow Bernanke, Gertler, and Watson (1997) by using two R^2 measures of fit. Denoting by $\widehat{y_{t|T}^+}$ the smoothed estimate of monthly GDP, and by $\widehat{u_{t|T}}$ the same estimate of the error term u_t, they consider:

$$\begin{aligned}
R^2_{\text{level}} &= \frac{\text{VAR}\left(\widehat{y^+_{t|T}}\right)}{\text{VAR}\left(\widehat{y^+_{t|T}}\right) + \text{VAR}\left(\widehat{u_{t|T}}\right)}, \text{ and,} \\
R^2_{\text{diff}} &= \frac{\text{VAR}\left(\widehat{\Delta y^+_{t|T}}\right)}{\text{VAR}\left(\widehat{\Delta y^+_{t|T}}\right) + \text{VAR}\left(\widehat{\Delta u_{t|T}}\right)}.
\end{aligned}$$

They claim it is more informative to report the R^2 in first differences since the same statistic in levels will always be close to unity.

We now adapt the state-space representation in (9) and (10) to the problem of back-casting a series which we observe part of its realizations but not all. In some sense, this is very close to the problem worked out in Mönch and Uhlig, since they only observe quarterly GDP for some but not all months of the year. Their solution was to set to zero the missing observations. This seems like a clever and natural solution. It shuts down the missing values of the observed quarterly series in monthly frequency that are used in forecasting the state variable. This same principle is applied here to construct back-cast estimates of employment and income for the Brazilian economy.

Suppose we posses a total of $t = 1, 2, \cdots, T^*, \cdots, T$, observations on x_t. However, for series y^+_t, we only posses data from $t = T^* + 1, \cdots, T$, with missing values from $t = 1, 2, \cdots, T^*$. This is exactly our setup for income and employment in this paper. If we set the order of the polynomial $\left(1 - \phi_1 L - \cdots - \phi_p L^p\right)$ to unity, i.e., $p = 1$, with coefficient ϕ, recalling that now we need not impose the time-aggregation restriction in (11), the state-space form of our problem collapses to the following:

$$\xi_t = \begin{pmatrix} y^+_t \\ u_t \end{pmatrix} = \begin{pmatrix} \phi & \rho \\ 0 & \rho \end{pmatrix} \begin{pmatrix} y^+_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} x_t\beta \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t \end{pmatrix} \tag{12}$$

$$y_t = \mathbf{H}'_t \xi_t, \tag{13}$$

where (12) and (13) are respectively the state and the observation equations and the matrix $\mathbf{H}'_t$ is time-varying, with the following format:

$$\mathbf{H}'_t = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix}, & t = T^* + 1, \cdots, T \\ \begin{bmatrix} 0 & 0 \end{bmatrix}, & \text{otherwise.} \end{cases} \tag{14}$$

The key to the problem lies in the choice for $\mathbf{H}_t'$ in (14). Here, we make the latent variable y_t^+ identical to y_t for the periods in which the latter is observed, with no error term. This has two consequences. First, the algorithm will forecast y_t^+ to be identical to y_t for $t = T^* + 1, \cdots, T$. Second, it will use the available data on employment (income) to estimate a model and will use this model to forecast the latent variable in the periods in which it is not observable, i.e., from $t = 1, 2, \cdots, T^*$. Under correct specification, this model can produce the optimal forecasts of the latent variable consistent with all available future information. That will be simply given by the smoothed forecast of y_t^+, i.e., by $\widehat{y_{t|T}^+}$.

4 Empirical Results

4.1 Data

An important part of this paper is the choice of the variables to be included in the coincident indicator. We follow the recent Brazilian experience: Duarte, Issler and Spacov (2004) and Spacov (2001). For output, labelled Y_t, we use industrial production, computed by IBGE, and available from 1980:1. There is not a long-span sales series in Brazil, we therefore follow Duarte, Issler and Spacov and use total Brazilian production of corrugated paper as a proxy for sales, labelled S_t, which is computed by ABPO. Employment, labelled N_t, is given by the total number of persons – 10 years old or older – that have a job. It is extracted from the *Monthly Employment Survey* computed by IBGE. Income is proxied by the labor income series, labelled I_t, extracted from this same *Survey.*

The last two series – employment and income – are only available from 2003 on, because of a drastic redesign of the *Monthly Employment Survey.* Here, we back-cast these series using a state-space representation estimated using the kalman filter.

4.2 The Coincident Series

As stressed above, one of the original contributions of this paper is to back-cast two of the coincident series for the Brazilian economy – income and employment. We used the techniques described in the previous section to back-cast them. In the current *Monthly Employment Survey*, income is available from 2002:2 on, while employment is available from 2002:3 on.

Back-casting was conducted in two steps. First we select the co-variate series, which could potentially explain the variations of income or employment. These co-variates are then used in the state-space regression, which is estimated using the framework described above – based on the algorithm by Mönch and Uhlig (2005). Our setup allows for several different dynamic models to be estimated, all described in Table 1, depending on different values for the parameters ϕ and ρ.

We tested seven series as auxiliary regressors in the back-casting procedure, all available for the period 1980:1 to 2007:11. They are: industrial production, output in the process industry, corrugated paper production, car production, steel production, cement production, energy production, and the monthly real GDP series estimated by Issler and Notini (2008). The dependent variables and all co-variates entered in levels in the state space representation, which is estimated in all the six different versions described in Table 1. In addition to the co-variates listed above, our models also include eleven seasonal dummies. In Table 2, we present the R^2_{diff} measure of fit for each model described in Table1.

Table 2 – Employment and Income Resulting $R^2_{\text{diff}}$for each Model

Model	Employment	Income
Static model in levels with IID residuals	0.4979	0.1134
Static model in levels with $AR(1)$ residuals	0.4729	0.0425
Static model in 1st differences with IID residuals	0.0072	0.0000
Dynamic model in levels with IID residuals	0.0597	0.0827
Dynamic model in 1st differences with IID residuals	0.0000	0.0000
Dynamic model in levels with $AR(1)$ residuals	0.0000	0.0048

Our final choice of auxiliary variables and models was as follows. For employment (in logarithms) we choose only the monthly GDP series and energy production (in logarithms) as co-variates. For income (in logarithms), we selected only the paper production series and cement production (in logarithms) as auxiliary variables. In both cases, the model with the highest R^2_{level} and R^2_{diff} was the static model with i.i.d. errors, where set the parameters ϕ and ρ equal to zero.

All four coincident series used in this paper are plotted below, which includes the results of the back-casted series. All four series – Production (Y_t), Sales (S_t), Income (I_t) and Employment (N_t) – were seasonally adjusted using the X-12 procedure. For income and employment, the shaded areas in the graphs below depict the actual sample in which we observe them.

Figure 1: Industrial Production - In log and Seasonally Adjusted

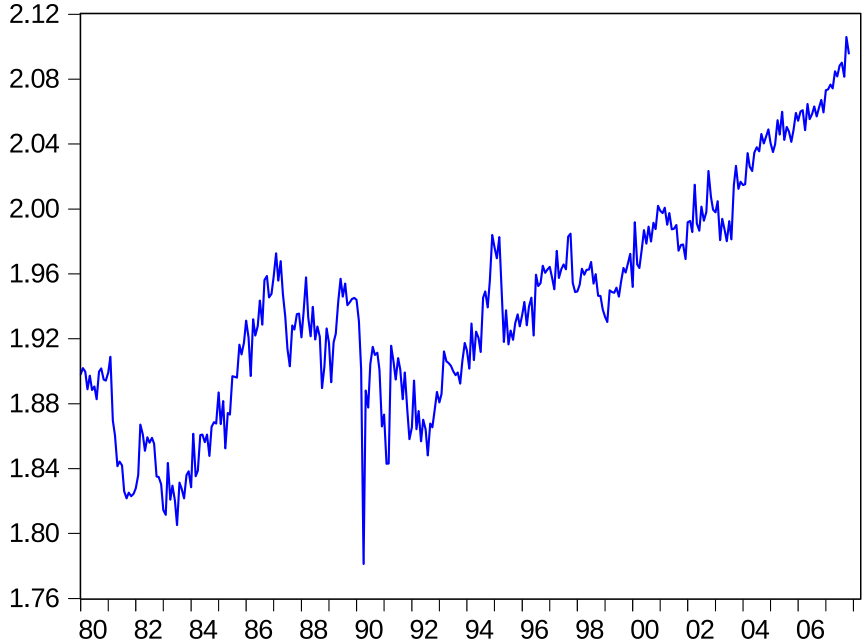

2.12
2.08
2.04
2.00
1.96
1.92
1.88
1.84
1.80
1.76
80
82
84
86
88
90
92
94
96
98
00
02
04
06

Figure 2: Sales - In log and Seasonally Adjusted

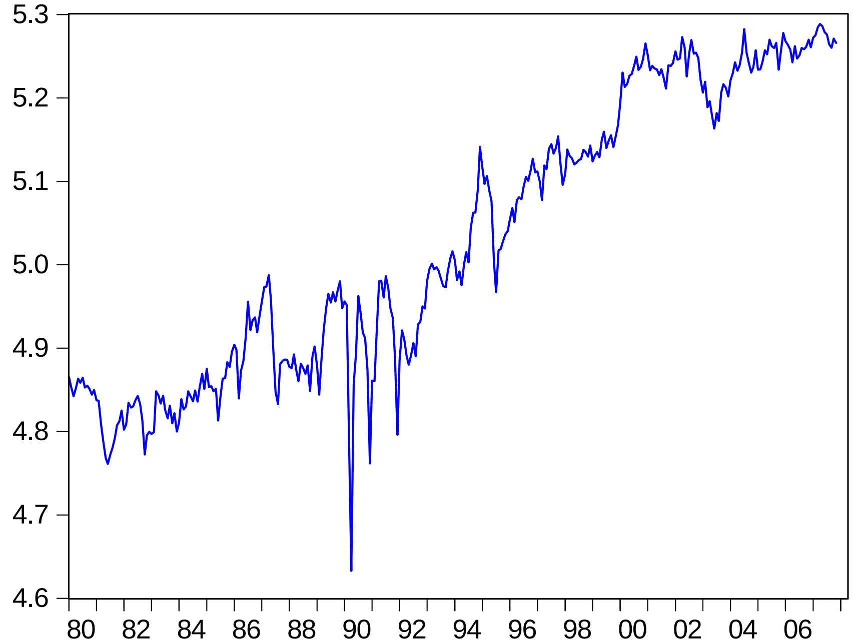
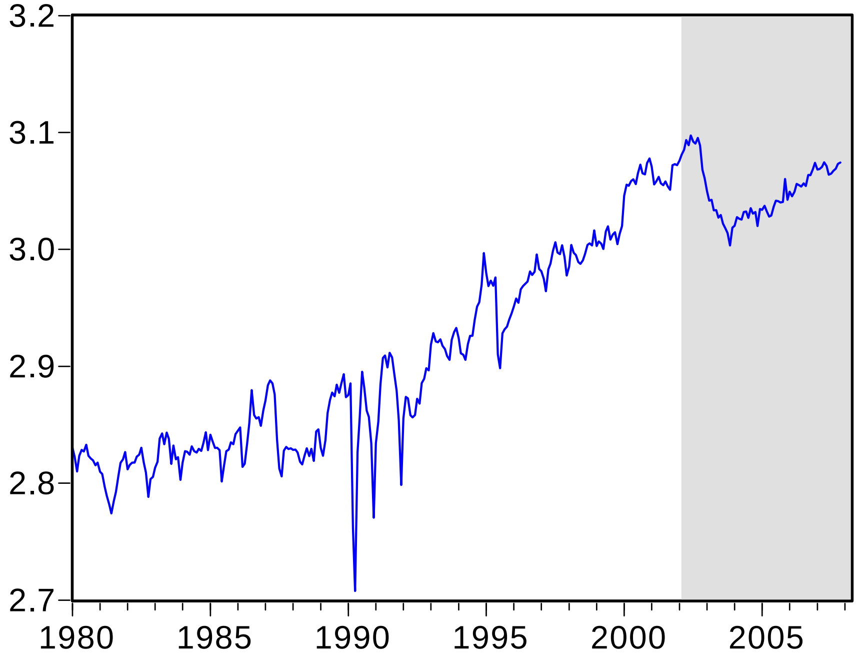
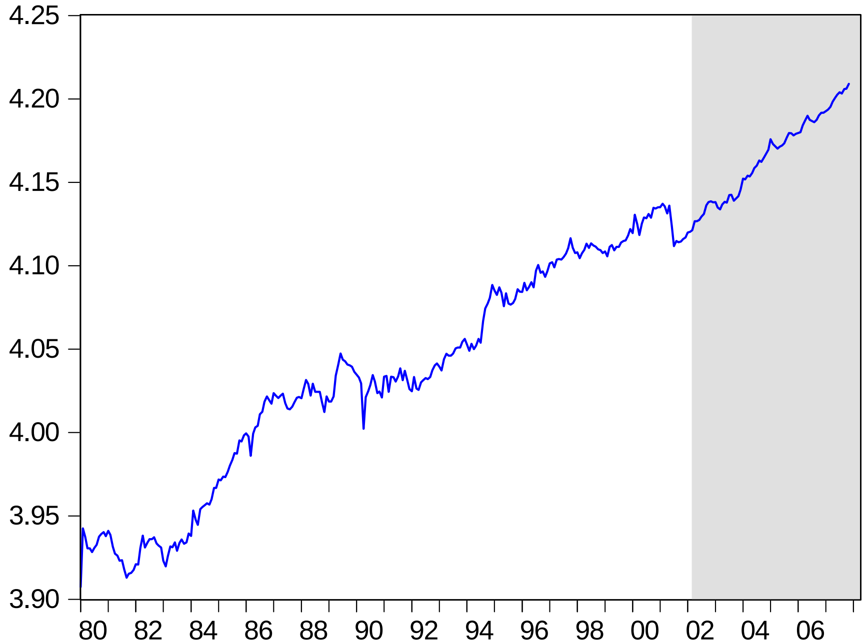


Figure 3: Income - In log and Seasonally Adjusted



Shaded areas depicts the actual sample

Figure 4: Employment - In log and Seasonally Adjusted



Shaded areas depicts the actual sample

All four coincident series were tested for unit roots. We used three different tests. On a preliminary basis, we used the Augmented Dickey-Fuller (ADF) test. Initial results were later examined in light of the results of the Phillips and Perron (1988) test and the stationarity test proposed by Kwiatkowski et al. (1992). All four coincident series showed signs of unit roots in testing and therefore were transformed into first differences (logs) prior to combination into a composite index.

Table 3: Coincident Series - Unit Root Tests

Variable	ADF		Kwiatkowski et. al	Phillips and Perron	
	t-statistic	p-value	LM-statistic	t-statistic	p-value
Employment	-0.62	0.86	2.13*	-0.55	0.88
Ind. Production	-0.49	0.89	1.78*	-0.87	0.80
Sales	-0.34	0.92	2.12*	-0.76	0.82
Income	-0.43	0.90	2.10*	-0.75	0.83

Notes:(i) ADF and Phillips and Perron H_0:series has a unit root; Kwiatkowski H_0:series is stationary.(ii)the asterisk (*) indicates that we reject the null hypothesis at 5%.

4.3 TCB's Coincident Index – $TCB - CI_t$

Using (1), we constructed a coincident index consistent with TCB's method, labelled $TCB - CI_t$, and plotted below. Next, we compare the turning-point dating of this index with that of two other indices: a monthly estimate of Brazilian GDP computed by Issler and Notini (2008) and the composite index previously proposed by Duarte, Issler and Spacov (2004), available until 2002:11. The latter also uses TCB's technique.

The turning points of these three composite indices were then compared using the Bry and Boschan (1971) and the Mönch and Uhlig (2005) dating algorithm, the latter being a slightly modified version of the former. Results in Table 4 show that the current dating using TCB's method yields results closer to the dating in Duarte, Issler and Spacov than to the dating of Brazilian monthly GDP. The most striking differences appear in the dating of the 1991 recession. The dating of Duarte, Issler and Spacov and of GDP encompass two recession episodes into one as compared to the dating of $TCB - CI_t$. It is also noteworthy that GDP misses the two last recessions as dated by $TCB - CI_t$ and by Duarte, Issler and Spacov's[1].

[1] This behavior – GDP missing the last two recessions – vanishes if one uses the modified Bry-Boschan dating method proposed in Mönch and Uhlig (2005) to date all three indices.

Figure 5: Coincident Index – Shaded Bry-Boschan Turning Points

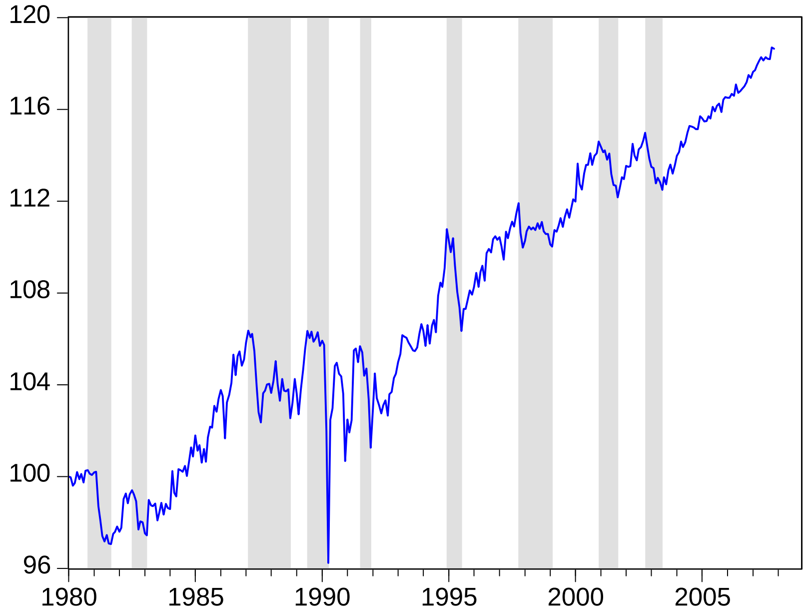


Table 4 – Turning-Point Comparisons Using Bry-Boschan Dating

Peak Dates			Through Dates		
$TCB-CI_t$	Duarte et al.	Brazilian GDP	$TCB-CI_t$	Duarte et al.	Brazilian GDP
1980:10	NA		1981:09	NA	1981:11
1982:07		1982:6	1983:02	1983:10	1983:02
1987:02	1987:04	1988:3	1988:10	1989:02	1988:10
1989:06	1989:08	1989:6	1990:04		
1991:07			1991:12	1991:03	1991:12
1994:12	1995:03	1994:12	1995:07	1995:09	1995:07
1997:10	1997:10	1997:10	1999:02	1999:02	1999:01
2000:12			2001:09		
2002:10	2002:4		2003:06		

Notes: The analysis in Duarte et al. (2004) starts in 1982:05, therefore could not have dated the recession of 1980. Brazilian GDP dating uses the monthly series constructed by Issler and Notini (2008).

Given the results in Table 4, we can compute how frequent Brazilian recessions are. From 1980-2007:11 we have a total of 9 recessions. On average, we observe in this period one recession at approximately every 3 years and 3 months, which is substantially more frequent than the U.S. historical average of one recession about every 5 years. Recessions in Brazil also last longer than U.S. recessions: while ours last about 12 months, on average, U.S. recessions last typically from 6 months to one year, on average (in our sample period here – 1980:1 to 2007:11 – U.S. recessions lasted, on average, 9 months). Indeed, Duarte, Issler and Spacov make the point that this behavior may be due to hardships that the Brazilian economy has endured in the post-1980 era, where GDP growth declined form about 7% a year in real terms prior to 1980 to about 2.2% a year after 1980.

Table 5 below lists Brazilian recessions from 1980:1 to 2007:11 when the dating of turning points is made using the modified Bry and Boschan technique proposed by Mönch and Uhlig (2005). The latter takes into account asymmetry differences in peak and through dating, which may be at work to explain the difference in dating between the Bry and Boschan and the Mönch and Uhlig method. Here, the dating of peaks in $TCB - CI_t$ is identical to that in Brazilian GDP, whereas the dating of throughs is almost identical.

Table 5 – Turning-Point Comparisons Using Mönch and Uhlig Dating

Peak Dates			Through Dates		
$TCB - CI_t$	Duarte et al.	Brazilian GDP	$TCB - CI_t$	Duarte et al.	Brazilian GDP
1980:10	NA	1980:10	1981:09	NA	1981:11
1982:07		1982:07	1983:02		1983:02
1987:02	1987:04	1987:02	1988:10	1989:02	1988:10
1989:06	1989:08	1989:06	1990:04		1990:04
1991:07		1991:07	1991:12	1991:12	1991:12
1994:12	1994:12	1994:12	1995:07	1995:9	1995:07
1997:10	1997:10	1997:10	1999:02	1999:02	1999:01
2000:12	2000:12	2000:12	2001:09	2001:9	2001:09
2002:10		2002:10	2003:06		2003:03

Notes: The analysis in Duarte et al. (2004) starts in 1982:05, therefore could not have dated the recession of 1980. Brazilian GDP dating uses the monthly series constructed by Issler and Notini (2008).

Taking into account the overall results of the dating exercise shows that the back-casting of income and employment proposed in this paper has the following properties: (i) generates sensible results for those series in the back-cast period; (ii) generates a sensible composite coincident index of economic activity. The latter is able to approximate reasonably well the turning points of monthly GDP and those of the TCB index using the retired income and employment series in Duarte, Issler and Spacov (2004). Of course, there are more similarities in turning-point dating when dating uses the technique proposed by Mönch and Uhlig.

We believe that the strategy we chose in this paper to construct a long span time-series for the Brazilian coincident indicator was the best possible. An alternative would be to chain the current employment and income series with their respective series retired by IBGE. Since the redesign of the *Monthly Employment Survey* was drastic, this procedure would chain completely different series. Another alternative would be to only use industrial production and sales to construct the composite index up to 2002:2, and then use the four usual series from 2002:3 onwards. This procedure would probably induce structural changes in mean and variance of the composite index after 2002:3.

4.4 The Composite Leading Indicator

Leading indicators are widely used in predicting turning points of business cycles in many countries. The selection of a leading indicator index involves three steps: (i) select an appropriate indicator as a measure of economic activity to be targeted, also called a reference series; (ii) select appropriate economic and financial indicators as predictors of the turning points of the reference series; (iii) combine the selected leading series in order to construct a composite leading index.

The first step was accomplished in the previous section, where we obtained a composite index of economic activity for the Brazilian economy after we back-cast the employment and income series. The next step is to select appropriate leading indicators as predictors of turning points. We search for series that satisfy the following conditions: (a) to be observable at a monthly frequency for the period 1980-2007:11; (b) timely data releases, and having small revisions regarding final data figures.

Recent research has shown that business-tendency survey data are particularly suitable for business cycle monitoring and forecasting. Business tendency surveys are conducted in all OECD member countries and have proved to be a cost-effective means of generating timely information on short-term economic fluctuations. In Brazil, the Brazilian Institute of Economics (IBRE) of Getulio Vargas Foundation (FGV) is a pioneering institution that computes surveys of economic activity. These include a survey of consumer expectations and another on business expectations on industrial production and related series: inflow of new orders, level of book orders, stocks of finished goods, etc.

From FGV's survey series and other Brazilian databases (IBGE, IPEADATA, and the Central Bank's), we selected 44 series that are candidates of being leading series of the coincident index. Our choice was guided by the international experience (Stock and Watson (1989, 1993)) and also by local experience (Duarte, Issler and Spacov (2004)).

A main issue regarding FGV's survey series is that they were computed on a quarterly frequency up to September 2005. From then on, surveys were then conducted on a monthly basis. Therefore, there is the need to interpolate the data on quarterly frequency to have an homogeneous series on a monthly basis. Our interpolation method was, again, Mönch and Uhlig's (2005).

All leading nominal series were deflated to reflect their purchasing power as of March, 2008. The deflator used was the Brazilian General Price Index "IGP-DI" – calculated by FGV. All series denominated in foreign currency were converted into Brazilian Reais at the prevailing exchange rate and subsequently deflated. All series were logged, unless logs could not be taken of the original series (potentially zero or negative figures). All series were also seasonally adjusted prior to the analysis using the X-12 procedure, whenever a seasonal pattern in them was detected.

With the exception of the survey-tendency series, all leading series were tested for unit roots. Survey series are bounded series, by construction. Therefore, they cannot posses a unit root, which leads to unbounded series in theory. To test for unit roots we used the Augmented Dickey-Fuller (ADF) test, the Phillips and Perron (1988) test, and the stationarity test proposed by Kwiatkowski et al. (1992). All series with a unit root were transformed into first differences (logs) prior to combination into a composite index[2].

In order to measure the quality with which a leading series correctly anticipates the "state of the economy" implied by the coincident series (recession or expansion), we use a criterion originally proposed by Diebold and Rudebusch (1999), and later employed by Zhang and Zhuang (2002) and Gallardo and Pedersen (1997). The Quadratic Probability Score, labelled as $QPS\,(h)$, is given by:

$$QPS(h) = \frac{\sum_{t=1}^{T} (P_t - R_t)^2}{T} \tag{15}$$

where P_t denotes the predicted state outcomes from a candidate leading indicator and R_t denotes the observed realizations of the reference series. Both are equal to one

[2] ADF unit-root test results are presented in Table A3 in the Appendix. Other test results are available upon request.

for a turning point and zero otherwise; T is the total number of sample observations, while h is the horizon in which the leading series potentially predicts the reference series. By construction, the value of $QPS(h)$ ranges between zero and one, with zero indicating a perfect fit for the the "state of the economy" of the reference series.

Next, we describe the basic criteria used to select the leading series that will compose our index. First, for each series, we calculate the optimum (minimum) $QPS(h)$ value, denoted by $QPS\,(h^*)$, where h^* is the resulting optimum lag. To be a leading series candidate, the series must have $h^* > 0$ in $QPS\,(h^*)$. This means that the series is leading, not lagging or coincident to reference series. Second, we apply Granger (1969) causality tests in order to examine whether the leading series precedes the reference series. We expect that a leading series Granger-causes the reference series but is not Granger-caused by it.

In the Appendix, Table A4 shows the $QPS\,(h^*)$, h^*, and Granger-causality test results. The majority of the potential leading series do not Granger cause the coincident series. The exceptions are some FGV's survey series, in addition to SELIC – Central Bank's basic interest rate – and IBOVESPA – Brazilian Stock Market Index. From them, IBOVESPA shows promise, since its $QPS\,(h^*) = 24.5\%$, and $h^* = 5$. This means that, when we take the IBOVESPA index, with a lag of 5 months vis-à-vis period t, it correctly predicts 24.5% of the "state of the economy" as measured by the peak and though behavior of our our composite index. A slightly worse result in observed to the survey series on the production of real-estate inputs – $QPS\,(h^*) = 25.1\%$, and $h^* = 1$.

The $QPS\,(\cdot)$ statistic has only three series with values between 10 and 20% – intermediate-good production, consumer-good production, and inventories, and a few between 20 and 30%. The intersection of the two criteria above – "Granger causality" and "low $QPS\,(h^*)$" – only has the IBOVESPA index and the production of real-estate inputs. Across all potential leading series the mean lag is 3, but the median and modal lag are 1. There are several interesting series which have $h^* > 1$ and a relatively low $QPS\,(h^*)$: FGV's survey series on inventories ($QPS\,(h^*) = 17.6\%$), the IBOVESPA index ($QPS\,(h^*) = 24.5\%$), as well as a myriad of other FGV's survey series.

Looking at results in Table A4, there is no obvious way to select series to be in the composite index. We present next 10 *ad-hoc* criteria to select those series, the idea being that we want h^* to be high, $QPS\,(h^*)$ to be low, and that a leading series Granger-causes the reference series but is not Granger-caused by it. We also investigate whether a series that has low h^*, with low $QPS\,(h^*)$, would also have a relatively low $QPS\,(h)$ for higher values of h. The 10 criteria are listed below:

1. Select all series possessing $QPS(h)$ less than 0.4 and positive optimum lag;

2. Select all series possessing $QPS(h)$ less than 0.4;

3. Select all series that satisfied the Granger causality test criterion;

4. Select all series in the intersection between the first and third criterion;

5. Select all series in the intersection between the second and third criterion;

6. Select all Survey series that satisfied the Granger causality test criterion;

7. Select the five series in Table A4 that have the lowest $QPS(h)$ value;

8. Select the series for which h^* is between two and seven months and $QPS\,(h^*) < 0.3$;

9. Select the series for which h^* is between two and seven months;

10. Select survey series for which $QPS\,(h^*) < 0.3$.

Given these criteria, we computed 10 different composite leading indices of economic activity, labelled $LI_{i,t}$, $i = 1, 2, \cdots, 10$. We chose to combine leading series into the composite index using a counterpart of equation (1) – equal weights on standardized growth rates of the leading series[3].

Table 6, below, lists the values of QPS for each criterion listed above, computed for the optimum lag, i.e., $QPS\,(h^*)$.

Table 6 – Leading Indices: $QPS\,(h^*)$ computed using Mönch and Uhlig's Method			
Series	Description	$QPS\,(h^*)$	h^*
$LI_{1,t}$	$h^* > 0$ and $QPS < 0.4$	0.1910	1
$LI_{2,t}$	$QPS < 0.4$	0.1612	1
$LI_{3,t}$	Series that Granger cause the Coincident Index	0.2478	1
$LI_{4,t}$	Granger cause, $h^* > 0$ and $QPS < 0.4$	0.2358	3
$LI_{5,t}$	Granger cause and $QPS < 0.4$	0.2328	1
$LI_{6,t}$	Granger cause (FGV survey series only)	0.2657	1
$LI_{7,t}$	The five series which have the lowest QPS	0.1015	1
$LI_{8,t}$	h^* between 2 and 7 and $QPS \leq 0.3$	0.2507	4
$LI_{9,t}$	h^* between 2 and 7 (FGV survey series only)	0.2866	3
$LI_{10,t}$	$QPS \leq 0.3$ (FGV survey series only)	0.2507	3

[3] Tables A5 and A6 in Appendix compare the turning points data for each leading index and the turning points of the coincident index.

From the results in Table 6 $LI_{7,t}$ stands out as a candidate of composite index. The leading series in it have their $QPS\,(h^{*})$ between 11.04% and 23.28%. Despite that, the composite index has a $QPS\,(h^{*}) = 10.15\%$ – lower than the smallest $QPS\,(h^{*})$ of the series in it. The latter are all Industry Survey series: of the Consumer-Good Industry, Capital-Good, Real-Estate Input, Intermediary-Good and the Level of External Demand. The composite indices $LI_{2,t}$ and $LI_{1,t}$ also do well in terms of $QPS\,(h^{*})$ and can be considered as an alternative to $LI_{7,t}$.

Our next exercise is a dating exercise involving $TCB - CI_t$ and $LI_{i,t}$, $i = 1, 2, \cdots, 10$. We want to examine how well and how often these leading indices predict the turning points in $TCB - CI_t$. We are also interested in knowing whether they generate false predictions, i.e., predicting a non-existent peak or through in economic activity. We start with a 24-month window around period t, i.e., from $t - 12$ through $t + 12$, and consider turning points in $TCB - CI_t$ and in $LI_{i,t}$, $i = 1, 2, \cdots, 10$. From peak and through dates in $TCB - CI_t$ and $LI_{i,t}$, we are able to match peaks of $TCB - CI_t$ with peaks of $LI_{i,t}$, and throughs of $TCB - CI_t$ with throughs of $LI_{i,t}$. We can also compute the average lead in peak (or through) prediction for each episode, as well as to list false predictions of turning points.

Results of this exercise are presented in Tables 7 through 11 for $LI_{7,t}$, $LI_{2,t}$ and $LI_{1,t}$. The Appendix contains this exercise for the remaining leading indices.

Table 7 shows respectively the coincident index and $LI_{7,t}$ peaks and through dates. Peak prediction is much better done than through prediction: only one peak is lost and $LI_{7,t}$ anticipates the coincident-index peaks 2.5 months ahead, on average. For throughs, although none is lost, on three occasions through prediction of $LI_{7,t}$ occurs after the through itself, reflecting on an average lead of 0.33 months for through prediction.

Table 7 - Turning Points Comparisons					
Mönch and Uhlig (2005) Dates					
Peak Dates			Through Dates		
$TCB - CI_t$	$LI_{7,t}$	Lead	$TCB - CI_t$	$LI_{7,t}$	Lead
1980:10			1981:09	1981:09	0
1982:07	1982:03	4	1983:02	1983:06	-4
1987:02	1987:01	1	1988:10	1988:09	1
1989:06	1989:05	1	1990:04	1990:03	1
1991:07	1991:03	4	1991:12	1992:02	-2
1994:12	1994:11	1	1995:07	1995:06	1
1997:10	1997:03	7	1999:02	1999:01	1
2000:12	2000:11	1	2001:09	2001:09	0
2002:10	2002:09	1	2003:06	2003:07	-1

Table 8 performs the same analysis above for $LI_{1,t}$. The average lead for for peak prediction is again 2.5 months, while that for through prediction is 2.25 months. However, $LI_{1,t}$ predicts two extra peaks and three extra throughs than those observed on $TCB - CI_t$. This result is in contrast with that of $LI_{7,t}$, which predicted no extra peaks or throughs.

For $LI_{2,t}$ the results in Table 9 show an average lead for peak prediction of 1.88 months, with a lead of -0.75 months for through prediction, a very bad result for through prediction. Moreover, $LI_{2,t}$ predicts one extra peak and two extra throughs than those observed on $TCB - CI_t$. This result is in contrast with that of $LI_{7,t}$, which predicted no extra peaks or throughs.

Focusing on the overall results for turning-point prediction only, it is clear that $LI_{7,t}$ dominates either $LI_{1,t}$ or $LI_{2,t}$: all three missed one peak, but $LI_{7,t}$ predicted no extra peaks, while $LI_{2,t}$ predicted two extra peaks and $LI_{2,t}$ predicted one. Regarding throughs, all three composite indices did not miss any, while $LI_{7,t}$ predicted no extra throughs, which contrasts with the results for $LI_{1,t}$ and $LI_{2,t}$: three and two, respectively.

Table 8 - Turning-Point Comparisons					
Mönch and Uhlig Dates					
Peak Dates			Through Dates		
$TCB-CI_t$	$LI_{1,t}$	Lead	$TCB-CI_t$	$LI_{1,t}$	Lead
1980:10			1981:09	1981:04	5
1982:07	1982:02	5	1983:02	1982:09	5
	1984:07			1985:03	
1987:02	1986:09	5		1987:06	
1989:06	1989:05	1	1988:10		
1991:07	1991:06	1	1990:04	1990:03	1
1994:12	1994:11	1	1991:12	1991:11	1
1997:10	1997:09	1	1995:07	1995:06	1
2000:12	2000:07	5	1999:02	1998:09	5
2002:10	2002:09	1	2001:09	2001:09	0
	2004:06		2003:06	2003:06	0
				2005:1	

Table 9 - Turning-Point Comparisons					
Mönch and Uhlig Dates					
Peak Dates			Through Dates		
$TCB-CI_t$	$LI_{2,t}$	Lead	$TCB-CI_t$	$LI_{2,t}$	Lead
1980:10			1981:09	1981:08	1
1982:07	1982:02	5	1983:02	1983:06	-4
1987:02	1986:09	5		1987:06	
1989:06	1989:05	1	1988:10		
1991:07	1991:06	1	1990:04	1990:03	1
1994:12	1994:11	1	1991:12	1992:07	-7
1997:10	1997:09	1	1995:07	1995:06	1
2000:12	2000:12	0	1999:02	1998:12	2
2002:10	2002:09	1	2001:09	2001:09	0
	2004:08		2003:06	2003:06	0
				2005:01	

Tables 10 and 11 contain, respectively, peak and through dating statistics for all 10 composite leading indices. It becomes clear that the good $QPS(\cdot)$ statistic for $LI_{7,t}$ is a consequence of not predicting extra throughs and throughs and not missing extra peaks and throughs vis-à-vis alternative indices.

All and all, considering the whole evidence in this section, we choose $LI_{7,t}$ to be our composite leading index of economic activity. Our choice is supported by a QPS value of 10.15%, meaning that this leading index provides wrong predictions of the state of the Brazilian economy only in 10.15% of the time.

There is a somewhat asymmetric behavior for $LI_{7,t}$ in terms of peak and through prediction: on average, while $LI_{7,t}$ predicts peaks with a two-and-a-half-month lead, it predicts throughs with a very small average lead of 0.37 months. Because of this behavior, we might want to consider either $LI_{1,t}$ as an alternative composite index for the purpose of through prediction only, since it leads $TCB-CI_t$ by 2.25 months.

Table 10 - Leading Indices: Peak Dating Comparisons

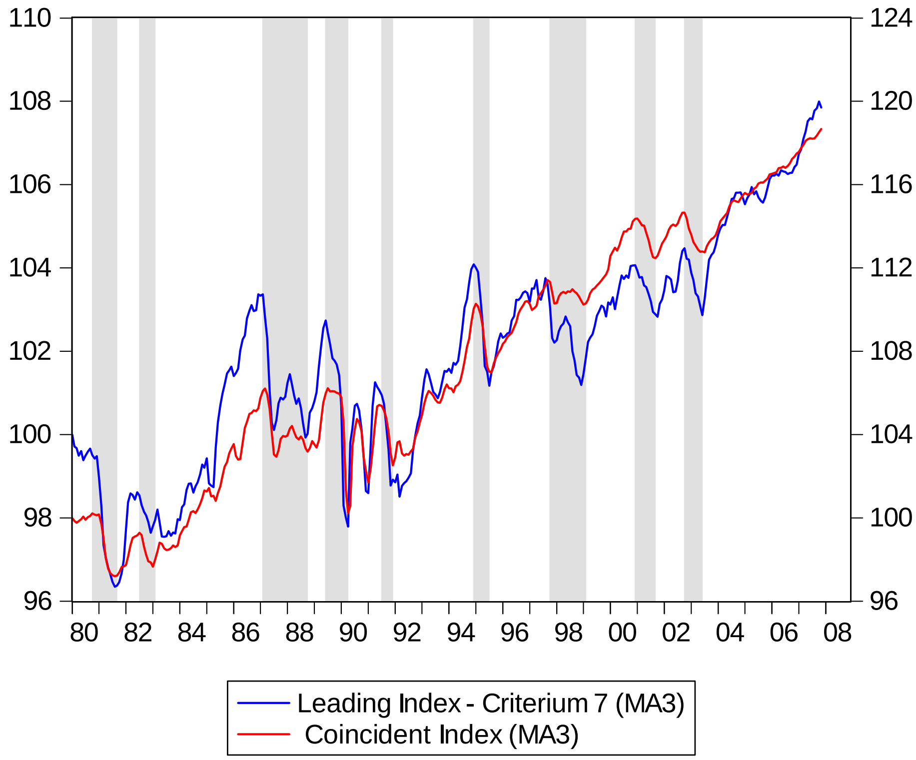
Mönch and Uhlig Dates

Index	# of Peaks	# Leading Peaks	# Missed Peaks	# Extra Peaks
$TCB-CI_t$	9	-	-	-
$LI_{1,t}$	10	8	1	2
$LI_{2,t}$	9	8	1	1
$LI_{3,t}$	7	6	3	1
$LI_{4,t}$	10	7	2	3
$LI_{5,t}$	8	6	3	2
$LI_{6,t}$	8	6	3	2
$LI_{7,t}$	8	8	1	0
$LI_{8,t}$	8	8	1	0
$LI_{9,t}$	10	7	2	3
$LI_{10,t}$	9	8	1	1

Table 11 - Leading Indices: Through Dating Comparisons				
Mönch and Uhlig Dates				
Index	# of Throughs	# Leading Throughs	# Missed Throughs	# Extra Throughs
$TCB - CI_t$	9	-	-	-
$LI_{1,t}$	11	8	1	3
$LI_{2,t}$	10	8	1	2
$LI_{3,t}$	8	7	2	1
$LI_{4,t}$	11	9	0	2
$LI_{5,t}$	9	7	2	2
$LI_{6,t}$	8	6	3	2
$LI_{7,t}$	9	9	0	0
$LI_{8,t}$	10	8	1	2
$LI_{9,t}$	11	8	1	3
$LI_{10,t}$	10	8	1	2

Finally, in the Figure below, we plot $TCB{-}CI_t$ and $LI_{7,t}$ smoothed by computing a three-month moving average. They have a straking similar behavior for the sample period covered in this paper. Of course, the original $LI_{7,t}$ leads the original $TCB{-}CI_t$ by 2.5 months for peaks and by 0.33 months for throughs.

Figure 6: Coincident and Leading Indexes



5 Conclusion

This paper has three original contributions. First, by back-casting the usual current employment and income series for Brazil, we allow business-cycle research in Brazil to resume using TCB and NBER oriented methods, which proved valuable after Duarte, Issler and Spacov (2004). Indeed, the main challenge of Brazilian business-cycle research was to be able to form a long enough coincident index with the usual series used in TCB's method – industrial production, sales, income and employment. Here, we devoted a great deal of effort in reconstructing employment and income using a novel flexible state-space representation based on the interpolation method of Mönch and Uhlig (2005).

Once we obtained a long enough span of the usual series used in TCB's method, we compute a new composite coincident index of Brazilian economic activity. Its dating of recessions is compared with those in Duarte, Issler and Spacov and with those implied by the monthly GDP estimate computed by Issler and Notini (2008).

Our last contribution is to propose a composite leading index of economic activity to track our composite coincident index. This is an important topic here, since Brazilian research had focused mainly on the construction of coincident indices. After a wide empirical search, we settled for a composite index that predicts correctly the "state of the economy" (expansion vs. recession), measured by our coincident index, almost 90% of the time. It misses one peak in economic activity and no through, while predicting no extra peaks or throughs. Moreover, on average, it leads the coincident index by 2.5 months for peaks and by 0.33 months for throughs. For anticipating throughs alone, an alternative composite leading index increases this lead to 2.25 months.

Finally, it is worth stressing that our choice of leading composite index – $LI_{7,t}$ – uses only series contained in the survey of industrial activity conducted by FGV: Consumer-Good activity, Capital-Good activity, Real-Estate Input activity, Intermediary-Good activity and the Level of External Demand. Since the criterion to choose the series in $LI_{7,t}$ was based solely on the five best values for $QPS(\cdot)$, it is interesting to find that only survey series made the top-five spots on that list.

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6 Apendix

6.1 The Bry and Boschan (1971) Algorithm

BRY BOSCHAN PROCEDURE FOR PROGRAMMED DETERMINATION OF TURNING POINTS

I. Determination of extremes and substitution of values

II Determination of cycles in 12-month moving average (extremes replaced)

A. Identification of points higher (or lower) than 5 months on either side

B. Enforcement of alternation of turns by selecting highest of multiple peaked (or lowest of multiple troughs).

III Determination of corresponding turns in Spencer curve (extremes replaced).

A. Identification of highest (or lowest) value within $\pm$ 5 months of selected turn in 12-month moving average.

B. Enforcement of minimum cycle duration of 15 months by eliminating lowerpeaks and higher troughs of shorter cycles

IV Determination of corresponding turns in short- term moving average of 3 to 6 months, depending on MCD (months of cyclical dominance).

A. Identification of highest (or lowest) value within $\pm$ 5 months of selected turn in Spencer curve.

V. Determination of turning points in unsmoothed series

A. Identification of highest (or lowest) value within $\pm$ 4 months, or MCD term, whichever is larger, of selected turn in short-term moving average.

B. Elimination of turns within 6 months of beginning and end of series.

C. Elimination of peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end.

D. Elimination of cycles whose duration is less than 15 months.

E. Elimination of phases whose duration is less than 5 months.

VI. Statement of final turning points.

Source: Bry and Boschan (1971) page 21.

6.2 Additional Tables

Table A1: Leading Series

Series name	Description	Source
BASE_R	Monetary base	Bacen
SELIC_R	Selic interest rate	Bacen
M1_R	M1 money stock	Bacen
IBOV_R	Ibovespa index	Bovespa
EXP_PRECOS	Exports prices	Funcex
EXP_QUANTUM	Quantum of exports	Funcex
EXP_R	Exports (FOB)	Funcex
TTROCA	Terms of trade	Funcex
IMP_PRECOS	Imports prices	Funcex
IMP_QUANTUM	Quantum of imports	Funcex
IMP_R	Imports (FOB)	Funcex
CAMBIO_R	Exchange Rate	Bacen
NUCIFIESP	Manufacturing Industry	Fiesp
PROD_BC	Production - Consumer Goods	IBGE/PIM
PROD_BCD	Production - Consumer Durable	IBGE/PIM
PROD_BCND	Production - Consumption and Non Durable	IBGE/PIM
PROD_BI	Production - Intermediate Goods	IBGE/PIM
PROD_BK	Production - Capital Goods	IBGE/PIM
PRODINDT	Industrial production - processing industry	IBGE/PIM
PRODONI	Production - bus	IBGE/PIM
PRODVEI	Production - vehicles	Anfavea
PRODAUTO	Production - motors	Anfavea
PRODCAM	Production - trucks	Anfavea
SAL_R	Nominal Salary - industry	
PO	Staff employed - industry	Fiesp
HPP	Hours paid - industry	Fiesp
HTP	Hours worked in production - industry	Fiesp
ICMS_R		Confaz
INPC_R	National Consumer Price Index	
SPC		ACSP
IPA_R		FGV
FALENCIAS	Bankruptcy - Sao Paulo Capital	

Table A2: Survey Leading Series

Business Tendency Survey	Description	Source
NUCI_BR	Survey of Manufacturing Industry	FGV
NUCI_BC	Survey of Consumer Goods Industry	FGV
NUCI_BK	Survey of Capital Goods Industry	FGV
NUCI_MC	Survey of Construction Materials Industry	FGV
NUCI_BI	Survey of Intermediaries Goods Industry	FGV
DEMINT	Survey of Industry - Level of Internal Demand	FGV
DEMEX	Survey of Industry - Level of External Demand	FGV
DEMPREVINT	Survey of Industry - Internal Demand Forecast	FGV
DEMPREVEXT	Survey of Industry - External Demand Forecast	FGV
DEMGLOB	Survey of Industry - Level of Global Demand	FGV
DEMPREV	Survey of Industry - Global Demand Forecast	FGV
EMPPREV	Survey of Industry - Employment forecast	FGV
ESTOQUES	Survey of Industry - Level of Inventories	FGV
PRODPREV	Survey of Industry - Production Forecast	FGV

Table A3: Leading Series - ADF Unit Root Test

Series	t-statistic	p-value
BASE_R	-2.89	0.17
CAMBIO_R	-1.83	0.69
DEMGLOB	-2.92	0.16
DEMPREV	-4.42	0.00*
EMPPREV	-2.72	0.23
ESTOQUES	-5.06	0.00*
EXP_PRECOS	-0.16	0.99
EXP_QUANTUM	-2.71	0.23
EXP_R	-1.85	0.68
FALENCIAS	-2.38	0.39
HPP	-1.85	0.68
HTP	-1.99	0.61
IBOV_R	-3.45	0.04*
ICMS_R	-5.53	0.00*
IMP_PRECOS	-0.77	0.97
IMP_QUANTUM	-2.91	0.16
IMP_R	-2.61	0.28
INPC_R	-1.68	0.00*
IPA_R	-1.08	0.93
M1_R	-2.15	0.52
NUCI_BR	-3.46	0.04*
NUCIFIESP	-5.20	0.00*
PO	-1.11	0.92
PROD_BC	-2.96	0.14
PROD_BCD	-3.52	0.04*
PROD_BCND	-2.96	0.15
PROD_BI	-2.28	0.44
PROD_BK	-3.10	0.11
PRODAUTO	-6.14	0.00*

PRODCAM	-5.23	0.00*
PRODINDT	-2.54	0.31
PRODONI	-1.11	0.00*
PRODPREV	-3.29	0.07
PRODVEI	-7.12	0.00*
SAL_R	-3.44	0.04*
SELIC_R	-1.45	0.00*
SPC	-2.16	0.51
TTROCA	-4.28	0.00*
NUCI_BC	-2.40	0.14
NUCI_MC	-2.16	0.22
NUCI_BI	-2.88	0.04*
DeMINT	-3.04	0.03*
DEMEX	-5.24	0.00*
DEMPREVINT	-3.84	0.00*
DEMPREVEXT	-3.40	0.01*

Notes: (i) the specification of the test equation was chosen on the basis of the Schwartz Information Criterion; (ii) the asterisk (*) indicates that we reject the null hypothesis of a unit root at 5%.

Table A4: Leading Series - $QPS(h^*)$ and Granger Causality

Leading	**Optimum h^***	**Mín QPS-$QPS(h^*)$**	**Granger-Causes**
BASE_R	1	0.4567	B
DEMGLOB	3	0.2806	B
DEMPREV	4	0.2866	B
EXP_R	7	0.3642	N
EXP_QUANTUM	12	0.3104	N
HPP	1	0.4299	B
HTP	1	0.3940	N
IMP_R	1	0.3761	N
IPA_R	11	0.5164	N
M1_R	1	0.3373	C
NUCI_BC	1	0.3463	C
NUCI_BK	1	0.3134	C
NUCI_MC	1	0.2507	C
PO	1	0.4090	N
PRODAUTO	1	0.2149	B
PROD_BC	1	0.1254	B
PROD_BCND	1	0.2328	N
PROD_BI	1	0.1104	N
PROD_BK	1	0.3463	N
PRODINDT	1	0.3104	N
ESTOQUES	2	0.1761	B
IBOV_R	5	0.2448	C
ICMS_R	1	0.3134	B
INPC_R	5	0.4776	N
NUCI_BR	1	0.2746	B
NUCIFIESP	1	0.2478	B
PROD_BCD	1	0.2746	N
PROD_CAM	1	0.2627	N
PRODONI	1	0.4179	N

Notes: i) Statistics $QPS(h^*)$ and $h^*$are computed in accordance with the description of the equation (15) in the text. (ii) in the Granger causality test, the symbol C means that the leading series Granger-cause at least three out of four series that make up the coincident index with the reciprocal is not true. The symbol B means bi-directional causality in the Granger causality test. The symbol N indicates that the leading series not Granger cause the coincident series. The level of significance was set at 5% in these tests and the number of lags tested was set at 3, 6, 12. To compute the results of the Granger test it was considered the existence of causality in at least one of these lags.

Table A4 (continuation)

Leading	**Optimum** h^*	**Mín QPS-**$QPS(h^*)$	**Granger-Causes**
PRODPREV	3	0.2507	N
PRODVEI	1	0.3224	N
SAL_R	1	0.3761	N
NUCI_BI	1	0.3224	B
CAMBIO_R	12	0.6000	B
EXP_PRECOS	3	0.3881	N
IMP_PRECOS	10	0.5045	N
SELIC_R	11	0.5821	C
TTROCA	2	0.3642	N
DEMEXT	6	0.3164	N
DEMINT	2	0.2746	B
DEMPREVEXT	4	0.3463	N
DEMPREVINT	4	0.2716	B
IMP_QUANTUM	1	0.3343	N
LN_SPC	1	0.2537	B

Notes: i) Statistics $QPS(h^*)$ and $h^*$are computed in accordance with the description of the equation (15) in the text. (ii) in the Granger causality test, the symbol C means that the leading series Granger-cause at least three out of four series that make up the coincident index with the reciprocal is not true. The symbol B means bi-directional causality in the Granger causality test. The symbol N indicates that the leading series not Granger cause the coincident series. The level of significance was set at 5% in these tests and the number of lags tested was set at 3, 6, 12. To compute the results of the Granger test it was considered the existence of causality in at least one of these lags.

Table A5: Leading Indices - Mönch e Uhlig Dates

Table A5 – Selected Leading Index					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI3	Lag	TCB - CI	LI3	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5			
1989:06	1989:03	-3	1988:10	1988:12	+2
1991:07	1991:04	-3	1990:04	1990:03	-1
1994:12	1994:11	-1	1991:12	1992:07	+7
1997:10	1997:08	-2	1995:07	1995:09	+2
2000:12	2001:03	+3	1999:02	1998:11	-3
2002:10			2001:09	2002:03	+6
	2004:09		2003:06		
				2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI4	Lag	TCB - CI	LI4	Lag
1980:10			1981:09	1981:03	-6
1982:07	1981:11	-8	1983:02	1982:10	-4
	1984:04			1985:03	
1987:02	1986:09	-5	1988:10	1988:07	-3
1989:06	1989 3	-3	1990:04	1990:02	-2
	1990:06				
1991:07			1991:12	1992:12	+12
1994:12	1994:09	-3	1995:07	1995:07	0
1997:10	1997:06	-4	1999:02	1998:11	-3
2000:12	2000:12	0	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:01	-5
	2004:06			2005:03	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI6	Lag	TCB - CI	LI6	Lag
1980:10			1981:09		
1982:07	1982:04	-3	1983:02	1983:06	+4
				1987:06	
1987:02	1986:09	-5	1988:10		
	1988:03				
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:06	+6
1994:12	1994:12	0	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:06	-12
	2004:09			2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI8	Lag	TCB - CI	LI8	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:04	-3	1983:02	1983:07	+5
				1987:7	
1987:02	1986:07	-7	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:07	0	1991:12	1991:10	-2
1994:12	1994:10	-2	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
2000:12	2000:07	-5	2001:09	2001:07	-2
2002:10	2002:01	-9	2003:06	2003:07	+1
				2005:10	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI9	Lag	TCB - CI	LI9	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:01	-6	1983:02	1982:10	-4
				1985:03	
	1984:04			1987:07	
1987:02	1986:10	-4	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:04	-3	1991:12	1991:10	-2
1994:12	1994:10	-2	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
	1999:10				
2000:12			2001:09	2001:10	+1
2002:10	2002:10	0	2003:06	2003:07	+1
	2004:07			2005:12	

Table A5 – Selected Leading Index (continuation)					
Turning Points Comparatives– Mönch e Uhlig					
Peak dates			Through dates		
TCB - CI	LI10	Lag	TCB - CI	LI10	Lag
1980:10			1981:09	1981:07	-2
1982:07	1982:04	-3	1983:02	1983:07	+5
				1987:07	
1987:02	1986:10	-4	1988:10		
1989:06	1989:04	-2	1990:04	1990:04	0
1991:07	1991:07	0	1991:12	1992:01	+1
1994:12	1995:01	+1	1995:07	1995:07	0
1997:10	1996:10	-12	1999:02	1998:10	-4
2000:12	2000:07	-5	2001:09	2001:10	+1
2002:10	2002:04	-6	2003:06	2003:07	+1
	2004:07			2005:10	

Table A6: Leading Indices - Bry-Boschan Dates

Table A6 – Selected Leading Index					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI1	Lag	TCB - CI	LI1	Lag
1980:10			1981:09	1981:04	-5
1982:07	1982:02	-5	1983:02	1982:09	-5
1987:02	1986:09	-5	1988:10	1987:06	-4
1989:06	1989:05	-1	1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1991:11	-1
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1998:09	-5
2000:12	2001:02	+2	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:06	0
	2004:06			2005:10	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI2	Lag	TCB - CI	LI2	Lag
1980:10			1981:09	1981:08	-1
1982:07	1982:02	-5	1983:02	1983:06	+4
				1987:06	
1987:02	1986:09	-5	1988:10		
1989:06	1989:05	-1	1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1992:07	+7
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:06	0
	2004:08			2005:10	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI3	Lag	TCB - CI	LI3	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10		
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:07	+7
1994:12	1994:11	-1	1995:07	1995:09	+2
1997:10	1997:08	-2	1999:02	1998:11	-3
2000:12	2001:03	-9	2001:09		
2002:10			2003:06	2003:06	0
	2004:09			2005:12	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI4	Lag	TCB - CI	LI4	Lag
1980:10			1981:09	1981:03	-6
1982:07	1981:11	-8	1983:02	1982:10	-4
	1984:04			1985:03	
1987:02	1986:09	-5	1988:10		
1989:06			1990:04		
1991:07			1991:12	1992:12	+12
1994:12	1994:09	-3	1995:07	1995:07	0
1997:10	1997:06	-4	1999:02	1998:11	-3
2000:12	2000:12	0	2001:09	2001:09	0
2002:10			2003:06		
	2004:06			2005:03	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI5	Lag	TCB - CI	LI5	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10		
1989:06			1990:04	1990:03	-1
1991:07	1991:06	-1	1991:12	1992:06	+6
1994:12	1994:11	-1	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:6	-12
	2004:09			2005:03	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI6	Lag	TCB - CI	LI6	Lag
1980:10			1981:09		
1982:07			1983:02	1983:06	+4
1987:02	1986:09	-5	1988:10	1987:06	-8
	1988:03				
1989:06			1990:04	1990:03	-1
1991:07	1991:04	-3	1991:12	1992:06	+6
1994:12	1994:12	0	1995:07	1995:09	+2
1997:10	1997:06	-4	1999:02	1998:12	-2
2000:12	2000:12	0	2001:09		
2002:10			2003:06	2002:6	-12
	2004:09			2005:12	

Table A6 – Selected Leading Index (Continuation)					
Turning Points Comparatives – Bry and Boschan					
Peak dates			Through dates		
TCB - CI	LI7	Lag	TCB - CI	LI7	Lag
1980:10			1981:09	1981:09	0
1982:07	1982:03	-4	1983:02	1983:06	+4
1987:02	1987:01	-1	1988:10	1988:09	-1
1989:06	1989:05	-1	1990:04		
1991:07			1991:12	1992:02	+2
1994:12	1994:11	-1	1995:07	1995:06	-1
1997:10	1997:09	-1	1999:02	1999:01	-1
2000:12	2000:11	-1	2001:09	2001:09	0
2002:10	2002:09	-1	2003:06	2003:07	+1

Table A7 - Leading series that compound Leading Index number 1

DEMGLOB
DEMPREV
EXP_R
EXP_QUANTUM
IMP_R
M1_R
PRODAUTO
PROD_BC
PROD_BCND
PROD_BI
PROD_BK
PRODINDT
ESTOQUES
IBOV_R
ICMS_R
PROD_BCD
PRODPREV
EXP_PRECOS
TTROCA
DEMEX
DEMINT
DEMPREVEXT
DEMPREVINT
SPC

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The leading series chosen are the ones which the QPS took less than 0.4 and the maximum lag was greater than zero.

Table A8 - Leading series that compound Leading Index number 2

DEMGLOB
DEMPREV
EXP_R
EXP_QUANTUM
HTP
IMP_R
M1_R
NUCI_BC
NUCI_BK
NUCI_MC
PRODAUTO
PROD_BC
PROD_BCND
PROD_BI
PROD_BK
PRODINDT
ESTOQUES
IBOV_R
ICMS_R
NUCI_BR
NUCIFIESP
PROD_BCD
PROD_CAM
PRODPREV
PRODVEI

Table A8: (continuation)

SAL_R
NUCI_BI
EXP_PRECOS
TTROCA
DEMEXT
DEMINT
DEMPREVEXT
DEMPREVINT
IMP_QUANTUM
SPC

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The leading series chosen are the ones which the QPS took less than 0.4 and the maximum lag was greater than zero. The difference with the criterion 1 is choice of the optimum lag. In this case we exclude the zero lag from que optimum lag so the series show optimum lag above zero.

Table A9 - Leading series that compound Leading Index number 3

BASE_R
M1_R
NUCI_BC
NUCI_BK
NUCI_MC
IBOV_R
NUCI_BI
CAMBIO_R
SELIC_R
DEMEXT

Notes: (i) these series were selected by the Granger Causality Test criterion.(ii) we consider the causality test for the lag 3, 6 and 12. The series identified as causing the series of the index in any of these lags, was included in the composite leading index number 1.

Table A10 - Leading series that compound Leading Index number 4

M1_R
IBOV_R
DEMEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The select series were subject to the Granger causality test. (ii) The criterion is the intersection of the first and third criterion.

Table A11 - Leading series that compound Leading Index number 5

M1_R
NUCI_BC
NUCI_BK
NUCI_MC
IBOV_R
NUCI_BI
DEMEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. The select series were subject to the Granger causality test. (ii) The criterion is the intersection of the first and third criterion.

Table A12 - Leading series that compound Leading Index number 6

DEMGLOB
DEMPREV
IBOV_R
PRODPREV
DEMPREVINT

Notes: (i) Survey series which granger causes the coincident index.

Table A13 - Leading series that compound Leading Index number 7

PROD_BI
PROD_BC
ESTOQUES
PRODAUTO
PROD_BCND

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) The index is formed by the five series which shown the lowest QPS.

Table A14 - Leading series that compound Leading Index number 8

DEMGLOB
DEMPREV
IBOV
PRODPREV
DEMPREVINT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Series which the QPS ≤ 0.3 and the optimum lag is in the open interval (2,7).

Table A15 - Leading series that compound Leading Index number 9

DEMGLOB
DEMPREV
PRODPREV
DEMEX
DEMPREVINT
DEMPREVEXT

Notes: (i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Survey series which the optimum lag is in the open interval (2,7).

Table A16 - Leading series that compound Leading Index number 10

DEMGLOB
DEMPREV
ESTOQUES
NUCI_MC
PRODPREV
DEMINT
DEMPREVINT
NUCI_BR

(i) these series were selected after being subjected to the Monch-Uhlig routine. We compare the turning point dates of the leading series with the ones of the coincident index. (ii) Survey series which the QPS ≤ 0.3. (iii) with the exception of NUCI_MC these are the same series definied by criterion number 9.